MATH-3 TEST 2 (Unit 2 – 1.9, 1.10, 2.1-2.3, 2.6-2.8, 3.1,& modeling)

Sample

100 points NAME:______________

Show all work on the test. On graphs, you are expected to use your knowledge of shifting etc. as opposed to simply plotting points. On all graphs, you must label the coordinates of 2 points

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Fill in the blanks. (2 points each)

(1) is f(x) = $3x^3 - 2x$ even, odd, or neither? odd________________ What type of symmetry does it have if any? origin______

$f(-x) = 3(-x)^3 - 2(-x) = -3x^3 + 2x = -(3x^2-2x) = -f(x)$

(2) The slope of a line parallel to x+4y=9 is $-\frac{1}{4}$________________________

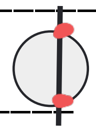
$4y = -x + 9$

$y = -\frac{1}{4}x + \frac{9}{4}$

(3) Is f(x) = $\frac{(x-2)^{3/2}}{x^4\sqrt{x+7}}$ a function? yes____________

(4) To obtain the graph of f(x) = $5+(x-2)^3$ we can shift the graph of g(x) = x^3 (how many units, which way?) 2 right, 5 up__________________________

(5) Is $x^2 + y^2 = 16$ an example of a function? No______________



fails vertical line test

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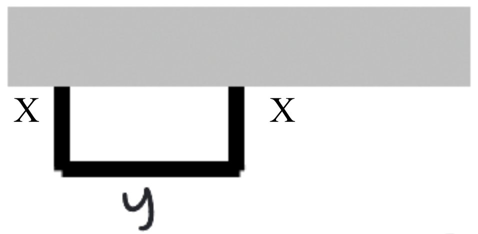
(6) A farmer has 120 feet of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. Find a function that models the area of the field in terms of one of the sides, X. What is the maximum area that can be enclosed in this way? (10 points)



$P = 120 \rightarrow 2x + y = 120$

$y = 120 - 2x$

$A = xy$

$A = x(120-2x)$

$A = 120x - 2x^2$

$A = -2x^2 + 120x$

vertex at $x = \frac{-b}{2a} = \frac{-120}{2(-2)} = 30$

parabola - opens down so max at vertex

$A = 30(120-60)$

$= 1800$ sq ft

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(7) Given f(x) = x^2 find and simplify: $\frac{f(x+h)-f(x)}{h}$ (6 points)

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \frac{2xh + h^2}{h} = 2x + h$$

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(8) Find the domain for each of the following functions (5 points each)

(a) f(x) = $\sqrt{\frac{1-x}{x+4}}$ Express answer as interval.

radicand must be ≥0 so

$\frac{1-x}{x+4} \geq 0$. Solve with signchart

− + −

-4 1

$(-4, 1]$

(b) g(x) = $\frac{\sqrt[3]{x-1}}{5x}$ ← No issues for root with odd index,

denom ≠ 0

⇒ $x \neq 0$

domain is $(-\infty, 0) \cup (0, \infty)$

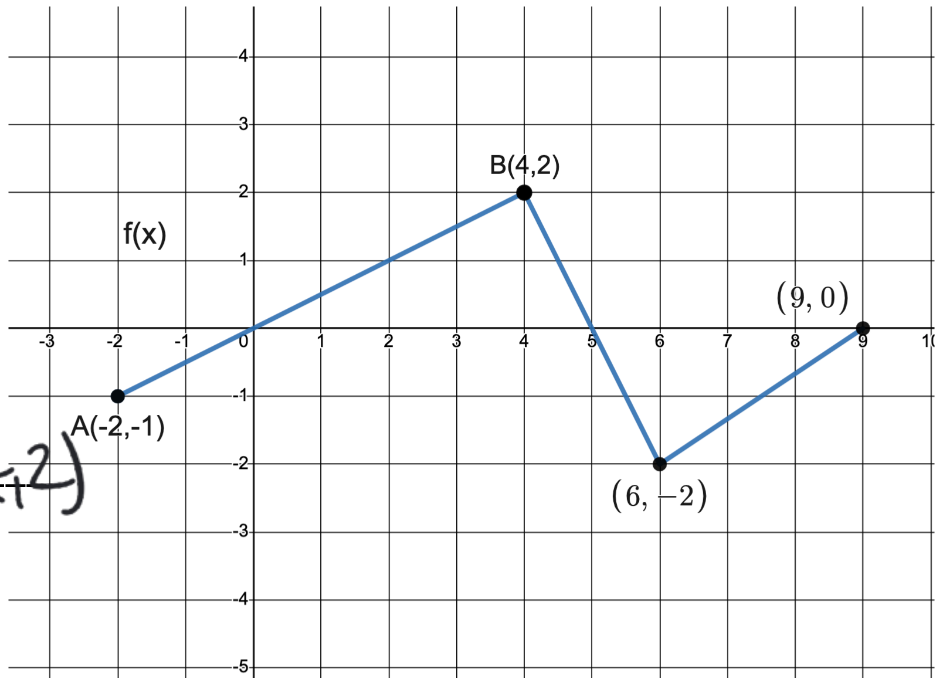
(9) Given the graph of $y = f(x)$ as shown (10 points)

(a) Express answers using interval notation:
-make it clear if you are using (vs. [

Domain of $f(x)$? $[-2, 9]$

Range of $f(x)$? $[-2, 2]$.

Where is $f(x)$ decreasing ? $(4, 6)$



(b) Find the coordinates of local max(s), if any $(4,2)$
(recall, local extrema do not occur at endpoints)

(c) What is the value of $f(-2)$? -1

(d) Find a value of a for which $f(a) = -2$ 6

(e) Is $f(x)$ a one-to-one function) no

(f) What is the slope of AB? $1/2$

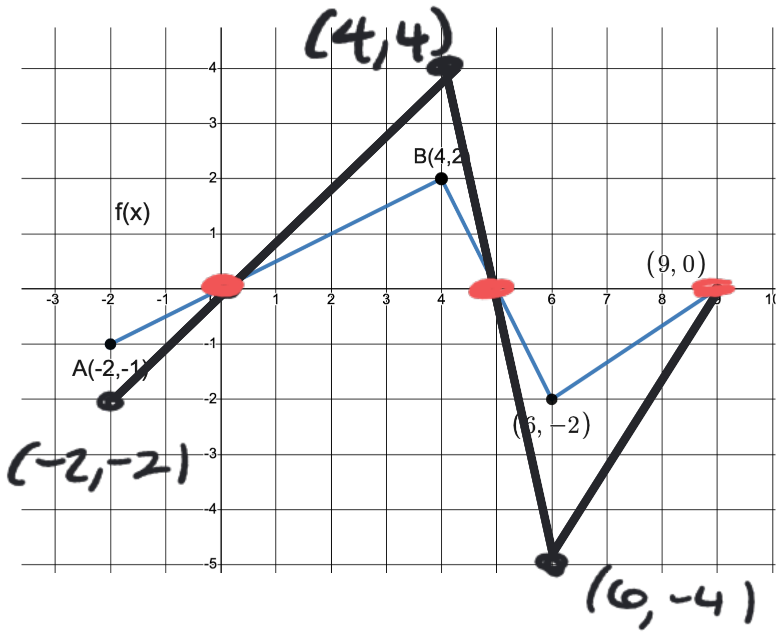
$m = \frac{2 - -1}{4 - -2} = \frac{3}{6}$

(g). What is the slope of a line perpendicular to AB? -2

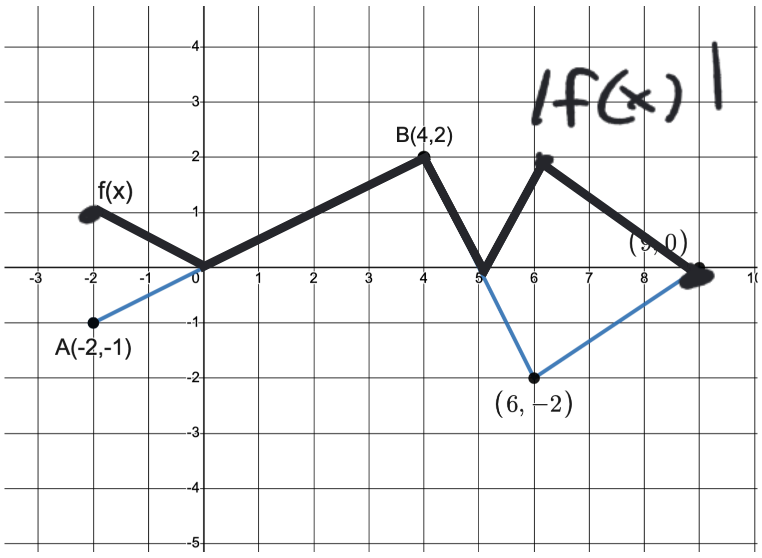
$\sqrt{(4 - -2)^2 + (3 - -1)^2} = \sqrt{36+16}$
$= \sqrt{52} = 2\sqrt{13}$

(h). What is the distance from A to B? $2\sqrt{13}$

(i) Use the graph of $f(x)$ shown at the right to graph $y = 2f(x)$

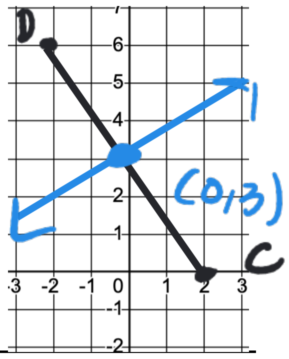


(j) Use the graph of $f(x)$ shown at the right to graph $y = |f(x)|$



(10) Find the equation of the line which is the perpendicular bisector of the segment C where C is (2,0) and D is (-2,6)

(8 points)



mid point of CD: $M = \left(\frac{2+-2}{2}, \frac{0+6}{2}\right)$

$= (0,3)$

slope of CD: $\frac{6}{-4} = \frac{-3}{2}$

slope of l : $2/3$

line :

$$\boxed{y = \frac{2}{3}x + 3}$$

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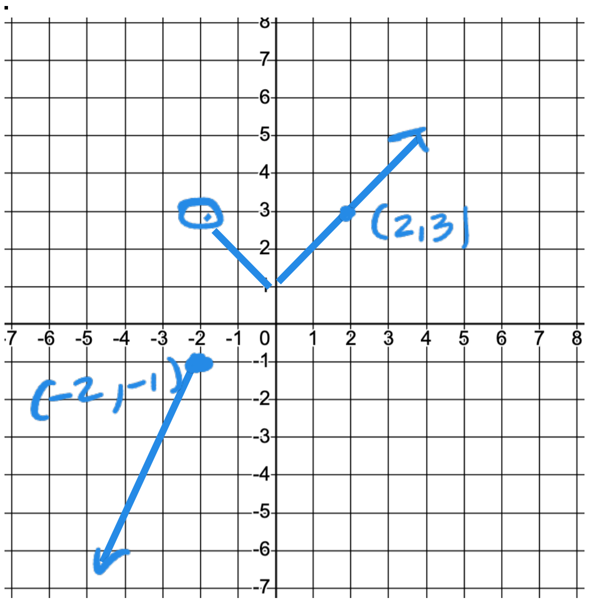
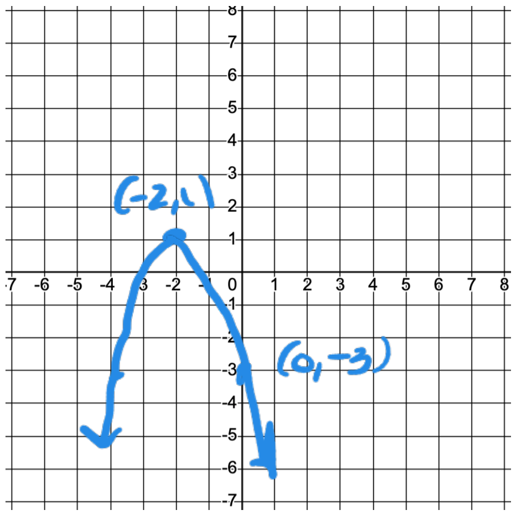
(11) Graph $\begin{cases} 2x+3 & if \;\; x \leq -2 \\ |x| + 1 & if \;\; x > -2 \end{cases}$ Show axes and scale.

(9 points)



(12) Graph f(x)= $-(x+2)^2 + 1$. (Explain how you used transformation to obtain this graph.

(9 points)



$y = x^2$

shift one left

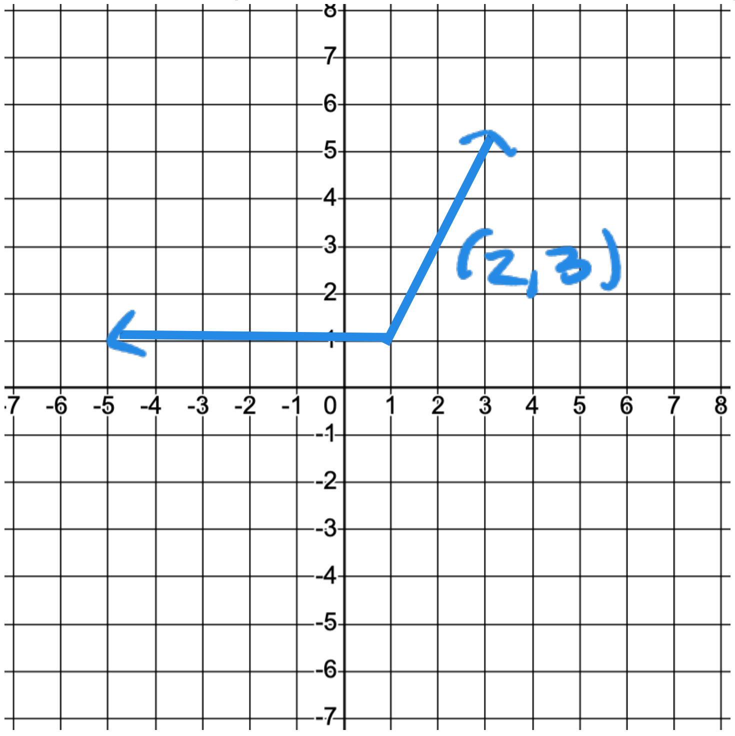
reflect across x axis

shift up

(13) f(x) = $|x-1|+x$ (5 points)

(a) Rewrite f as a piecewise defined function (i.e. how can we remove the bars)

(b) Graph f(x). Show scale and label 2 points on graph.



$$|x-1| = \begin{cases} x-1 \text{ if } x-1 \geq 0 & (x \geq 1) \\ -(x-1) \text{ if } x-1 < 0 & (x<1 \end{cases}$$

$$|x-1|+x = \begin{cases} x-1+x & \text{if } x \geq 1 \\ -(x-1)+x & \text{if } x<1 \end{cases}$$

$$= \begin{cases} 2x-1 & \text{if } x \geq 1 \\ 1 & \text{if } x<1 \end{cases}$$

(14). (a) Find the inverse of the function $f(x)=\sqrt{x-2}$. (Pay attention to any restrictions which must be made)

(b). Graph both $y=f(x)$ and $y=f^{-1}(x)$ on the same set of axes to verify they are inverses.

(c) What is the domain and range of $y=f(x)$ and $y=f^{-1}(x)$

(d). Verify it is the inverse by showing

$$(f \circ f^{-1})(x)=x$$

	f	f^{-1}
domain	$[2,\infty)$	$[0,\infty)$
range	$[0,\infty)$	$[2,\infty)$

Find $f^{-1}(x)$

$y=\sqrt{x-2}$

switch x,y

$x=\sqrt{y-2}$

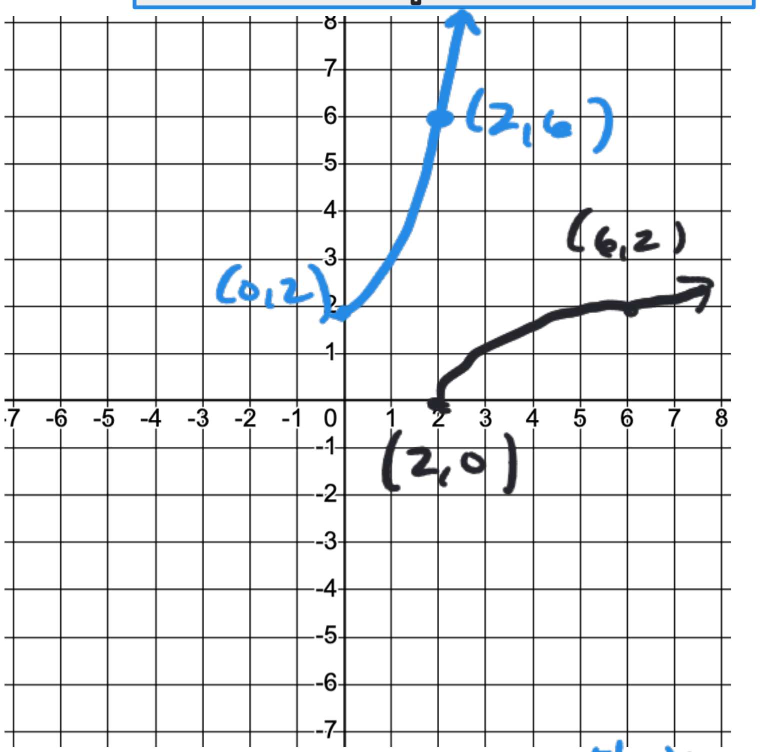
$x^2=y-2$

$y=x^2+2$

Must restrict $x \geq 0$

so $f^{-1}(x)$ is one-to-one

$f^{-1}(x)=x^2+2, x \geq 0$



$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f(x^2+2) \;;\; x \geq 0$$

$$= \sqrt{x^2+2-2} = \sqrt{x^2} = |x| \;;\; x \geq 0$$

$$= x \text{ since } x \geq 0$$